

Paper #4 Abstract

Title

PowerUp!-Mediator: Software for Designing Group-randomized Studies of Mediation

Authors

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Purpose

The purpose of this study is to disseminate the results of recent advances in statistical power analyses with regard to multilevel mediation and its implementation in the PowerUp!-Mediator software (Dong, Kelcey, Spybrook, & Maynard, 2016; Kelcey, Dong, Spybrook, & Shen, in review; Kelcey, Dong, Spybrook, & Cox, in review;). We first focus on the conceptual and statistical differences among common asymptotic, component-wise, and resampling-based tests of mediation as well as their performance in different contexts. We then introduce newly derived power formulas and delineate the statistical and substantive interpretation of the parameters that govern power in studies of multilevel mediation. Third, we outline reasonable values for these parameters across different education contexts using recent empirical compilations of values of these parameters. Finally, we demonstrate the use of the PowerUp!-Mediator software along with the formulas and parameter values to plan studies.

Background

Designs that facilitate inferences concerning both the total and indirect effects of a treatment potentially offer a more holistic description of interventions because they can complement ‘what works’ questions with the comprehensive study of the causal connections implied by substantive theories. Mapping the sensitivity of designs to detect these effects is of critical importance because it directly governs the types of evidence researchers can bring to bear on theories of action under realistic sample sizes. In this study, we review recent research deriving closed-form expressions to estimate the power to detect causally-defined individual, contextual, and cumulative indirect effects in two-level group-randomized studies. We unpack these formulas under the purview of typical multilevel mediation models and anchor their interpretation in the potential outcomes framework. The framework provides power analysis formulas and software that reduce calculations to simple functions of the primary path coefficients (e.g., treatment-mediator and mediator-outcome relationships) and common summary statistics (e.g., intraclass correlation coefficients). Probing these formulas suggests that group-randomized designs will typically be well-powered to detect individual indirect effects and can be well-powered to detect contextual and cumulative indirect effects when carefully planned.

Illustration

To briefly demonstrate the practical use of the formulas and software, let us consider a two-level study with students nested within schools. The goal of the study is to understand the extent to which a school-level program improves a student-level outcome through a student-level mediator. For instance, assume we are studying an intervention that has been randomly assigned to schools and is designed to improve student mental health (outcome) by improving student

knowledge of positive mental health behaviors (mediator). In addition, researchers suspect that a secondary pathway involves the intervention impacting the outcome through changes in the collective school-level knowledge base of students. As a result, the theory of action suggests that the intervention improves student mental health by both improving individual and collective student knowledge.

In planning this study, let us assume that we intend to sample 50 students per school (n_1), expect intraclass correlation coefficients for the outcome and mediator of 0.10 (ρ), and anticipate that the covariates collected will explain about 10% of the variance in the mediator at both levels and 25% of the variance in the outcome at both levels. Let us further assume that we anticipate a total or average treatment effect of 0.40—of which only a small 0.05 (12.5%) flows through the individual-level mediator while another 0.15 (37.5%) passes through the group-level mediator. Let us hypothesize that the decomposition of these indirect effects happens as follows: the projected difference in mediator values between treatment and control conditions is approximately 0.5 standard deviations (a), and the unique individual- (b_1) and group -level (b_2) mediator-outcome standardized associations are 0.10 and 0.30 respectively (and assume treatment-mediator interactions add/subtract 0.025 at both levels for the treatment/control group). That is, let $a=0.5$, $b_1=0.10$, $b_2=0.30$, $c'=0.20$, $\delta_1/2=\delta_2/2=0.025$, $R^2_{M_{\alpha_2}^{L_1}}=R^2_{M_{\alpha_2}^{L_2}}=0.10$, and $R^2_{Y_{\alpha_2}^{L_1}}=R^2_{Y_{\alpha_2}^{L_2}}=0.25$. Given these guidelines, we wish to identify a school-level sample size that would provide an 80% chance of detecting a total cross-level indirect effect as small as 0.0625 (0.5(0.10+0.025)) assuming a type one error rate of 0.05.

Figure 1 outlines the software module selection and parameter value input for power analyses for indirect effects in two-level cluster-randomized trials (i.e., cluster random assignment designs [CRA]). The top panel outlines the levels of the treatment, mediator, and outcome (e.g., 2-1-1 refers to a study in which the treatment has been assigned to clusters and the mediator and outcome are measured at the individual-level). In the bottom panel, users input the selected values for the parameters (to be discussed in detail). In turn, Figure 2 plots the power formulas tracking the power to detect the cross-level (individual) indirect effects, school-level (contextual) indirect effects and overall (individual and contextual) indirect effects.

In Figure 2, we display the power for the Sobel (solid curves), joint (dashed curves), and Monte Carlo interval tests (dotted curves) for the overall (black), school-level (light gray) and cross-level (dark gray) indirect effects as a function of group-level sample size (n_2). For the current example, our analyses suggest that sampling even just 18 schools (9 treatment, 9 control) would provide a sufficient power to detect the cross-level indirect effect. For the overall indirect effect we would need between 64 and 72 schools and for the school indirect effect we would need about 126 schools.

References

- Dong, N., Kelcey, B., Spybrook, J., & Maynard, R. A. (2016). PowerUp!-Mediator: A tool for calculating statistical power for causally-defined mediation in cluster randomized trials. (Version 0.4) [Software]. Available from <http://www.causalevaluation.org/>
- Kelcey, B., Dong, N., Spybrook, J., & Shen, Z. (in review). Statistical power for causally-defined group-level mediation in group-randomized studies. *Multivariate Behavioral Research*.
- Kelcey, B., Dong, N., Spybrook, J., & Cox, K. (in review). Statistical power for causally-defined individual and contextual indirect effects in group-randomized trials. *Journal of Educational and Behavioral Statistics*.

Figure 1

Software module selection and parameter value input

PowerUp!-Mediator to Detect Mediator Effects in 2-Level CRTs: Models and Corresponding Worksheets

Study Design	1	2	3	4																																																	
	Number of Total Levels of Clustering	Model Number	Level of Mediator	Power Calculation																																																	
<i>Cluster Random Assignment Designs (Level of Assignment \neq Level of Analysis)</i>																																																					
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Figure 2

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